Measuring Common and Market-Specific Information Flows

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Abstract

This study estimates the common and market-specific information flows when an asset is traded in multiple markets. We use a latent common factor to capture cross-market return correlations. The model produces unique estimates of the information shares that are unaffected by a market's trading speed. It avoids difficult econometric issues associated with ultra-high frequency sampling. At sampling intervals from 0.01 to 2 seconds, the common information shares are 66% to 94% across the listing and other exchanges, and are 59% to 90% between quote and trade prices. As trading speed increases, the common information shares have increased over time.

Keywords: Price discovery, common information, market-specific information, high frequency trading, information share.

JEL Classification: G10, G14, C32

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I. Introduction

Price discovery is an essential function of financial markets. When an asset is simultaneously traded in multiple markets, Hasbrouck (1995) provides the first model to compare price discovery across markets. With the proliferation of high-frequency trading (HFT), Hasbrouck (2021a) shows that measuring price discovery requires returns at ultra-high frequencies, e.g. 10 microseconds. In the Hasbrouck model, price discovery is allocated to individual markets, with the sum across all markets being 100%. In this study, we propose a model that allows common reaction to new information across markets, and estimate the common as well as market-specific information flows. We draw comparison with price discovery estimates in Hasbrouck (2021a), and show that the common information accounts for over 60% of the total information flow at intervals longer than 10 milliseconds.

The literature on measuring the information content of stock returns started with a series studies by Hasbrouck (1991a, 1991b, 1993). The models decompose stock returns into a random-walk component reflecting changes in expectation due to new information, and a serially-correlated noise component. The information content of stock prices is measured by the variance of the random-walk return component. Hasbrouck (1995) extends this information measure to multiple markets that trade the same asset: "A market's contribution to price discovery is its information share, defined as the proportion of the efficient price innovation variance that can be attributed to that market" (p1177). Hasbrouck (2002) provides further justification for the economic interpretation of the random-walk return, supporting the use of its variance as an information flow measure.

A key assumption of the Hasbrouck model is that all information, public and private, is priced in one of the markets first, and then propagates across markets. However, when returns are correlated contemporaneously across markets, it is difficult to attribute price innovations (information) to one particular market. Hasbrouck (1995) recommends shortening the sampling interval to reduce cross-market correlations, and using the triangularization of the covariance matrix to estimate the upper and lower bounds of the information shares. He estimated information shares of stock exchanges in 1993 using 1-second interval. Hasbrouck (2021a) shows that in 2016, the upper and lower bounds converge when returns are sampled at 10 microsecond intervals. Ultra-high frequency sampling helps to attribute return innovations to a particular market. It also creates difficult econometric issues, e.g. extremely long lags in the econometric model, millions of parameters to be estimated, high computation cost, and low numerical accuracy in estimating millions of miniscule changes.

Ultra-high frequency sampling creates two additional problems in measuring price discovery across markets. Suppose there are two markets A and B trading the same stock and a price-relevant public news is released, e.g. a report of a new COVID variant. Investors are racing to submit or revise orders. The first problem is that the initial price change in each market is unlikely to reflect the new information. The race is won by high-frequency trading (HFT) firms hitting stale orders. As argued by Budish et al. (2015), if there are N HFT firms with equal speed and one of them is trying to revise its quotes, the stale quote is hit with probability (N-1)/N. The stale bid is hit if it is bad news, and the stale ask is hit if it is good news. Therefore, the price impact of HFT is in the direction of the news, as shown by Broggard et al. (2014). But the first trade is at the stale price: the return does not reflect the new information.

The second problem is that price innovations in slower markets are partially attributed to the fastest market, even when all markets are reacting to the same news. If A is first hit at time t and B is hit at t+1, A is considered the price leader in the vector error correction model (VECM): B's price change is partially attributed to A through the return lead-lag relation. Price innovations in market B are underestimated. When sampling at ultra-high frequencies, price discovery becomes sequential across markets, and innovations in slower markets are underestimated. Hasbrouck (1995) observes that "the information share measures 'who moves first' in the process of price adjustment". Putnins (2013) reports that "the vast majority of the literature takes the view that a price series dominates price discovery if it is the first to adjust to new information about the fundamental value".

This study mitigates the above problems by reducing sampling frequency and allowing multiple markets to react to news in the same period. We decompose price innovations into a component that is common across markets and a market-specific component that is contemporaneously uncorrelated. Our approach offers three benefits. First, we estimate the common information flow extracted from the simultaneous price reactions in multiple markets. It captures the price impact of any price-relevant public signals, from macro and company announcements to high-frequency order flows and trades. It also captures the price impact of common order flows which may carry private information.[1] The separation of the common versus market-specific information shares is motivated by the observation that public information is the main source for price-relevant information, and different markets do react to public information almost simultaneously. It is also motivated by the cross-market price cointegration in the

[1] An example of common order flows is inter-market sweep orders (ISO) which are simultaneously routed to multiple markets to be filled against the displayed size. Chakravarty, et al. (2012) suggests that ISOs are used by informed institutional investors to split large orders into smaller ones for fast trading.

VECM, which implies that price innovations in different markets should have a large common component. The common information share provides a natural measure for the degree of information integration across markets at different time intervals. The comparison of the common vs market-specific information shares at different time intervals provides new insight into the price discovery process.

Second, by allowing for contemporaneous return correlations and estimating the common information, we avoid the need for ultra-high frequency sampling, and the bias in estimating information flow in favor of the fastest market. As discussed above, ultra-high frequency sampling implies that news is spread sequentially across markets and the VECM implies that slower markets learn from the price change in the fastest market, when all markets are simultaneously reacting to the same news. By using sampling intervals at least 1000 times of 10 microseconds, we allow multiple markets to change price in the same interval, perhaps even multiple times. The common component of the cross-market price changes is defined as the common information. After controlling the common information, information embedded in market-specific order flows can affect subsequent price changes in other markets. In our model, which market moves first is unimportant.

Third, our use of longer sampling intervals avoids the difficult econometric issues associated with ultra-high frequency sampling, e.g. Hasbrouck (2021a). At sampling intervals of 0.01 to 2 seconds, we can estimate all parameters of the model with very low computation cost. After accounting for the common return innovations, market-specific information shares are uniquely estimated without invoking Cholesky factorization. Our model with even longer intervals, e.g. daily, can be applied to any informationally linked markets, e.g. exchange rates against USD.

Our main empirical strategy is to draw comparison with Hasbrouck (2021a) where the common information share is zero. We estimate the information shares of IBM across the listing exchange (NYSE) and other exchanges from Oct 3 to Nov 11 in 2016. Our main finding is the high common information share. At sampling intervals of 0.01 second, the common information share between the listing and other exchanges is 66%, the listing exchange has 16% information share, while the other exchanges together account for 18%. The common information share increases to 80% at 0.1 second, 90% at 1 second, and 94% at 2 second intervals. The high common information shares reflect the high cross-market return correlations, which are 0.455, 0.617, 0.757, and 0.816 at intervals of 0.01, 0.1, 1, and 2 seconds respectively. Malceniece et al. (2019) show that HFT has sharply increased return correlations across different stocks, and the increase is mainly due to correlated trading strategies of HFT.

For IBM stock from Oct 3 to Nov 11, 2016, Hasbrouck (2021a, Table 7) reports a 66% information share of quotes and a 34% information share of trade prices at 10-microsecond intervals. At sampling intervals of 0.01, 0.1, 1, and 2 seconds, our estimates of the common information shares between quotes and trade prices are 58%, 74%, 85%, and 89% respectively. After accounting for common information, the quote information share ranges from 9% (2 seconds) to 23% (0.1 second) and the trade information share ranges from 1.1% (0.1 second) to 2.4% (2 seconds). When the model allows common information, trade price loses almost all of its information content but quote retains a significant portion of its information content. The dominance of quotes over trade prices, especially at high frequencies, is consistent with 14 times more quotes than trades; Hasbrouck (2021a, Table 2). This sharp difference in quotes and trades may also explain why at different sampling intervals, the common information shares between quotes and trade prices are consistently lower than the common information shares across different markets. Quotes are more evenly distributed across markets, with 33% on NYSE and 67% on other exchanges.

Beyond the comparison with Hasbrouck (2021a), we also estimate historical common information shares of IBM at 1-second intervals in 2000 and 2008. Since 2000, at least three factors have helped to increase cross-market return correlations, therefore the common information flow. The first is the rising trading speed that synchronizes prices at different exchanges. The second is the proliferation of trading platforms: IBM trading was concentrated on the NYSE in 2000 but spread more evenly across trading platforms. The third is the increased intensity of news arrivals, drawing attention and reactions from more investors. Indeed, we find that in 2000, the common information share is low at 11.8%, with the NYSE dominating the information share. HFT proliferated following the pass of Reg NMS in 2005. The common information share rises to 67% in 2008. The continued improvement in HFT technologies and strategies further increases the common information share to 90% in 2016.

The paper is organized as the following. Section II presents the econometric model and estimation procedure. It draws theoretical comparison between the proposed and the existing information share measures. Section III reports data and summary statistics. Section IV estimates the proposed information shares and draws empirical comparison with existing measures. Section V concludes.

II. Measuring common and market-specific information flows

This section presents our measures for common and market-specific information flows. We start with a summary on measuring information flow through a vector error correction model (VECM). A factor structure is then proposed to separate the VECM residuals into common and market-specific components. The common and market-specific information shares are jointly determined by the coefficients of the factor structure and the VECM residuals. We provide a detailed discussion on identification and inference, and compare the proposed information shares with the existing ones.

II.1 Measuring information flow

Suppose that an asset is simultaneously traded in n markets indexed by $i = 1, \ldots, n$. Let $p_t = [p_{1,t}, \cdots, p_{n,t}]'$ be the vector of log prices prevailing in the n markets at time t (the end of a period). Let $\mathcal{I}_t$ be the information set generated by $\{p_t, p_{t-1}, \ldots\}$. Let $\Delta p_t = p_t - p_{t-1}$ be the return vector for the time interval $(t-1, t]$. Since all prices are for the same asset at the same time t, p_t can be described as

(1) $$p_t = \iota_n m_t + u_t,$$

where ι_n is a n-dimensional vector of ones; m_t is a scalar martingale relative to $\mathcal{I}_t$ that represents the efficient (log) price; u_t is the n-dimensional stationary pricing noise that may correlates with the efficient price change Δm_t and may be autocorrelated. While m_t embodies the underlying value of the asset, the stationary noise u_t is transient such that p_t does not deviate from m_t for long. Typically in the literature of price discovery, attempts are made to decompose Δm_t into the contributions from n markets.

Following Hasbrouck (1995), we assume that the dynamics of the log prices in p_t can be described by a vector autoregressive (VAR) model: $A(L)p_t = \varepsilon_t$, where L is the lag operator; $A(L)$ is a $n \times n$ matrix polynomial in L with order K; the roots of $|A(z)| = 0$ are either outside the unit-circle or equal to one; and ε_t is a n-dimensional martingale difference (MD) process relative to $\mathcal{I}_t$ with variance matrix Ω. The law of one price implies that the log prices are cointegrated with the cointegrating matrix $\beta' = [\iota_{n-1}, -I_{n-1}]$, where I_{n-1} is the identity matrix of size $n-1$. Thus $\beta' p_t = \beta' u_t$ is stationary by (1). The VAR model $A(L)p_t = \varepsilon_t$ can be expressed as the vector error correction model (VECM):

(2) $$\Delta p_t - \Gamma_1 \Delta p_{t-1} - \cdots - \Gamma_{K-1} \Delta p_{t-K+1} - \alpha\beta' p_{t-1} = \varepsilon_t,$$

where $A(L) = \Phi(L)(1-L) - \alpha\beta' L$; $\Phi(L) = I_n - \Gamma_1 L - \cdots - \Gamma_{K-1} L^{K-1}$; $\Delta = 1 - L$; α is the adjustment matrix ($n \times (n-1)$) with full column rank; Γ_i are the coefficient matrices ($n \times n$). Here, ε_t represents price innovations in different markets. In general, the components in ε_t are correlated across markets and the variance matrix Ω is dense. Since β is known, the parameters $(\Gamma_1, \ldots, \Gamma_{K-1}, \alpha, \Omega)$ can easily be estimated by ordinary least squares.

In the VECM in (2), the log price vector p_t can be represented as a moving average of the reduced-form shock ε_t, known as the Beveridge-Nelson decomposition,

(3) $$p_t = \bar{p}_0 + \Psi \sum_{\tau=1}^{t} \varepsilon_\tau + u_t \qquad \text{with} \qquad u_t = \sum_{j=0}^{\infty} C_j \varepsilon_{t-j},$$

where $\bar{p}_0$ is the initial price; Ψ is a constant matrix; C_j exponentially tends to zero as j increases. Here, u_t is the pricing noise, as indicated in (1). Johansen (1995, Theorem 4.2) shows that

(4) $$\Psi = \beta_\perp[\alpha'_\perp \Phi(1)\beta_\perp]^{-1}\alpha'_\perp \, , \qquad \bar{p}_0 = \beta_\perp \bar{m}_0 \, ,$$

where $\bar{m}_0$ is a scaler dependent only on the information set at $t = 0$; $\beta_\perp$ is an orthogonal complement of β such that $\beta'_\perp\beta = 0$ and $[\beta, \beta_\perp]$ is invertible (similarly for $\alpha_\perp$ of α). Given that $\beta' = [\iota_{n-1}, -I_{n-1}]$, $\beta_\perp$ may simply be chosen as $\beta_\perp = \iota_n$. Further, $\alpha_\perp$ may be chosen as the eigen vector associated with the zero eigen value of $\alpha\alpha'$.

The vector of the log efficient price is defined as (see Lehmann (2002))

$$\iota_n m_t \equiv \lim_{j\to\infty} E(p_{t+j}|\mathcal{I}_t) = \bar{p}_0 + \Psi \textstyle\sum_{\tau=1}^{t} \varepsilon_\tau = \iota_n\{\bar{m}_0 + [\alpha'_\perp\Phi(1)\iota_n]^{-1}\alpha'_\perp \sum_{\tau=1}^{t}\varepsilon_\tau\}.$$

The law of one price implies that there is only one efficient price across all markets:

(5) $$m_t = \bar{m}_0 + [\alpha'_\perp\Phi(1)\iota_n]^{-1}\alpha'_\perp \textstyle\sum_{\tau=1}^{t}\varepsilon_\tau,$$

which underlies all prices in p_t. The efficient price change is given by

(6) $$\Delta m_t = m_t - m_{t-1} = [\alpha'_\perp\Phi(1)\iota_n]^{-1}\alpha'_\perp\varepsilon_t = h'\varepsilon_t,$$

where $h' = [h_1, \cdots, h_n] = [\alpha'_\perp\Phi(1)\iota_n]^{-1}\alpha'_\perp$. The total information flow across ALL markets is Var(Δm_t) estimated over a period, e.g. a trading day. If Ω is diagonal, i.e. $\varepsilon_{i,t}$ is uncorrelated with $\varepsilon_{j,t}$ for all $j \neq i$, Var(Δm_t) = $\sum_{i=1}^{n} h_i^2\sigma_i^2$ where σ_i^2 is the diagonal elements of Ω. In this case, the same information is priced sequentially across different markets. If markets react to the same information in the same period, the components in ε_t are correlated and the variance matrix Ω is dense, and it is difficult to attribute Var(Δm_t) to individual markets. Hasbrouck (1995, p1183) advocates the use of high-frequency data to enhance the sequential nature of price discovery across market. It would reduce cross-market return correlations, and maintain Var(Δm_t) $\approx \sum_{i=1}^{n} h_i^2\sigma_i^2$.

II.2 Common and idiosyncratic information shares

Our method to separate common versus idiosyncratic information follows closely to Yang (2017) who studies effects of macroeconomic shocks. We assume that return innovation in market i, $\varepsilon_{i,t}$, is the sum of a common shock that influences all markets, and an idiosyncratic shock that only affect market i contemporaneously. Therefore, the reduced-form shock to market i has the factor error structure

(7) $$\varepsilon_{i,t} = \delta_i f_t + \sigma_i\eta_{i,t}, \qquad i = 1, \dots, n,$$

where (δ_i, σ_i) are constant parameters; f_t is the common shock; $\eta_{i,t}$ is the idiosyncratic shock; f_t and $\eta_{i,t}$ are uncorrelated. Both f_t and $\eta_{i,t}$ are MD relative to $\mathcal{I}_t$ with variance one. The common shock f_t captures the effect of the latent public signals actioned by all markets in period t, or common order flows across markets driven by private information. The idiosyncratic shock $\eta_{i,t}$ represents the information embedded only in market i's order flows, because some investors may prefer to trade in a particular market with low fee or liquidity rebate. It is not contemporaneously correlated with returns in other markets.

The factor error structure can also be presented in matrix form

(8) $$\varepsilon_t = \delta f_t + \Sigma^{1/2}\eta_t,$$

where the common shock f_t is a scalar MD process with variance one; the idiosyncratic shock vector η_t is a MD process with variance I_n; $\Sigma = \mathrm{diag}[\sigma_1^2, \cdots, \sigma_n^2]$ and $\delta = [\delta_1, \ldots, \delta_n]'$; f_t and η_t are uncorrelated. The factor error structure (8) can be tested against data. Under (8), the correlations among the reduced-form shocks in ε_t are completely explained by the presence of f_t, and the variance of ε_t is constrained to be

(9) $$\Omega = \delta\delta' + \Sigma.$$

As Ω can be estimated from the reduced-form model (2), the restriction (9) allows us to test whether or not (8) fits data (see Section 2.4 below).

With the factor error structure, the efficient price change in (6) can be decomposed as

(10) $$\Delta m_t = [\alpha_\perp'\Phi(1)\iota_n]^{-1}\alpha_\perp'\varepsilon_t = h'\delta f_t + h'\Sigma^{1/2}\eta_t = h'\delta f_t + \textstyle\sum_{i=1}^n h_i\sigma_i\eta_{i,t},$$

where $h' = [h_1, \cdots, h_n] = [\alpha_\perp'\Phi(1)\iota_n]^{-1}\alpha_\perp'$, $h'\delta = \sum_{i=1}^n h_i\delta_i$ and $h'\Sigma^{1/2} = [h_1\sigma_1, \cdots, h_n\sigma_n]$. Clearly, $h'\delta$ and $h'\Sigma^{1/2}$ characterise how the efficient price change is influenced by the common shock f_t and idiosyncratic shocks in $\eta_t = [\eta_{1,t}, \ldots, \eta_{n,t}]'$ respectively. For instance, $h_1\sigma_1$ is the effect of the first market's idiosyncratic shock $\eta_{1,t}$ on the efficient price change while holding the prices of other markets fixed. Since the shocks in η_t and f_t are uncorrelated, the variance of the efficient price change can be unambiguously decomposed into the contributions of the common shock and idiosyncratic shocks

$$\mathrm{var}(\Delta m_t) = (h'\delta)^2 + h'\Sigma h = (h'\delta)^2 + \textstyle\sum_{i=1}^n h_i^2\sigma_i^2.$$

The contribution of the common shock to $\mathrm{var}(\Delta m_t)$, termed the common information share (CIS), is

(11) $$CIS = \frac{(h'\delta)^2}{(h'\delta)^2 + \sum_{i=1}^n h_i^2\sigma_i^2}.$$

The contribution of market i to $\mathrm{var}(\Delta m_t)$, termed the market-specific information share (MIS), is

(12) $\quad MIS_i = \frac{h_i^2 \sigma_i^2}{(h'\delta)^2 + \sum_{j=1}^{n} h_j^2 \sigma_j^2}, \qquad i = 1, \dots, n.$

Clearly $\sum_{\mathrm{i=1}}^{\mathrm{M}} \mathrm{MIS_i}$ + CIS = 1. The diagonal structure of Σ in (8) makes it easy to estimate market i's idiosyncratic information flow and information share. We do not need to invoke Cholesky decomposition to estimate idiosyncratic market information share.

In summary, our information shares are based on two testable assumptions: (i) the log price vector p_t fits the VECM model (2); (ii) the reduced-form error ε_t fits the factor error structure (8). When these assumptions are met, our information shares have clean interpretations. We now consider the identification and inference of (h, δ, σ_i^2).

II.3 Identification and inference

As h in (10) depends only on the parameters of the reduced-form VECM in (2), its estimation is straightforward. The $2n$ parameters in (10) can potentially be solved from (9), where the symmetric matrix Ω has $n(n+1)/2$ free elements that are estimable from (2). Therefore, heuristically, δ and Σ can be recovered from Ω when $n(n+1)/2 \geq 2n$, i.e. $n \geq 3$. However, comparing the number of parameters against the number of equations in (9) is not sufficient for identifying δ and Σ. We rely on Anderson and Rubin (1956, Theorem 5.5) for identification, which is given below and tailored for our notation.

Theorem 1 (Anderson and Rubin). *The parameters in* (δ, Σ) *in* (8) *are locally identifiable if and only if* (i) $n \geq 3$*;* (ii) *there are at least three non-zero elements in* δ.

Here, the meaning of "locally identifiable" is that the parameters in (δ, Σ) is uniquely identified only in a neighbourhood of the true parameter point, because there is always a "remote" parameter point $(-\delta, \Sigma)$ that is observationally equivalent to (δ, Σ). For our information measures in (11) and (12), this indeterminacy is immaterial as δ_i is in squared form. Given that the parameters $[\sigma_1^2, \dots, \sigma_n^2]$ can be any non-negative values, this theorem also covers the cases where some elements of Δp_t do not subject to idiosyncratic shocks (when $\sigma_i^2 = 0$ for some i).

Other restrictions can also be imposed on (δ, Σ). For example, an interesting restriction is that the elements in δ are identical: $\delta_1 = \cdots = \delta_n$, or the impact of the common shock on each market is the same. This is particularly useful for the case $n = 2$, where conditions in Theorem 1 are not met, and the additional restriction $\delta_1 = \delta_2$ can be imposed to achieve identification.

We use the maximum Gaussian quasi likelihood to estimate the parameters of the model defined in (2) and (8). The log quasi likelihood is

(13) $\ell_T = -\frac{Tn}{2}\ln(2\pi) - \frac{T}{2}\ln|\delta\delta' + \Sigma| - \frac{1}{2}\sum_{t=1}^{T}[A(L)p_t]'(\delta\delta' + \Sigma)^{-1}[A(L)p_t],$

with T being the sample size. As the cointegrating matrix β is known, the maximisation can be equivalently done in two steps: (i) run the OLS on (2) to estimate $A(L)$ and produce the residuals $\hat{\varepsilon}_t = \hat{A}(L)y_t$; (ii) estimate $[\Sigma, \delta]$ by maximising the concentrated log quasi likelihood

(14) $\ell_T(\Sigma, \delta) = -\frac{Tn}{2}\ln(2\pi) - \frac{T}{2}\ln|\delta\delta' + \Sigma| - \frac{1}{2}\sum_{t=1}^{T}\hat{\varepsilon}_t'(\delta\delta' + \Sigma)^{-1}\hat{\varepsilon}_t.$

When the conditional mean in (2) and the variance in (9) are correctly specified, and the conditions of Theorem 1 are met, the quasi maximum likelihood estimators $[\hat{A}(L), \hat{\Sigma}, \hat{\delta}]$ are consistent and asymptotically normal; see Bollerslev and Woolridge (1992).

The parameters (Σ, δ) are over-identified for $n > 3$, as there are more equations in (9) than the parameters at the right-hand side of (9). This feature makes it possible to test whether or not data fit in the factor error structure (8) by using an over-identification test. With the null hypothesis being (9), the test can be implemented by the likelihood ratio statistic

(15) $LR_T = 2[\ell_T(\hat{\Omega}) - \ell_T(\hat{\Sigma}, \hat{\delta})]$

where $\ell_T(\hat{\Omega})$ is the maximised log likelihood of the reduced-form VECM in (2), which is the unrestricted model; and $\ell_T(\hat{\Sigma}, \hat{\delta})$ is the maximised value of (14). Under mild conditions, LR_T has an asymptotic null distribution of χ^2 with $n(n+1)/2 - 2n = n(n-3)/2$ degrees of freedom when ε_t is normally distributed (see Gourieroux and Monfort (1989, Vol.2, p107)). Given that the normality of ε_t is not guaranteed in practice, the AIC and SIC information criteria may also be used to assess the adequacy of the factor error structure. Specifically, the criterion difference between the reduced-form model (2) and the factor error structure (8) is

(16) $d_T = LR_T - k_T(n-3)/2,$

where $k_T = 2$ for AIC and $k_T = \ln T$ for SIC. The factor error structure is not rejected by the information criterion if $d_T \leq 0$.

For $n = 3$, the null distribution for the test in (15) is not applicable. In this case, when the parameters in (8) are identified under the conditions of Theorem 1, they can be estimated from (14). Further, if (8) is the true data generating process, the parameters can also be solved from (9). Hence, if the parameters cannot be solved from estimated Ω in (9), then (8) must be a mis-specification. The following theorem details conditions under which the factor error structure (8) is compatible with the reduced-form variance Ω (hence a correct specification) when $n = 3$.

Theorem 2. *Consider the case of* $n = 3$*. Let the elements in a positive definite matrix Ω be ω_{ij} for* $i, j = 1,2,3$*. Then* $\delta = [\delta_1, \delta_2, \delta_3]'$ *and* $\Sigma = \mathrm{diag}[\sigma_1^2, \sigma_2^2, \sigma_3^2]$ *can be solved from* (9)*, expressible as functions of* ω_{ij}*, and unique up to a sign change for δ, if and only if* (i) $\omega_{12}\omega_{13}\omega_{23} > 0$*;* (ii) $\omega_{11} \geq \omega_{12}\omega_{13}/\omega_{23}$*,* $\omega_{22} \geq \omega_{12}\omega_{23}/\omega_{13}$*, and* $\omega_{33} \geq \omega_{23}\omega_{13}/\omega_{12}$.

Proof of Theorem 2. First, (9) can be written as

$$\omega_{ii} = \delta_i^2 + \sigma_i^2, \quad \omega_{ij} = \delta_i \delta_j, \qquad i, j = 1,2,3, \quad j > i.$$

These imply that $\delta_1^2 = \omega_{12}\omega_{13}/\omega_{23}$ and $\sigma_1^2 = \omega_{11} - \delta_1^2$, which hold if and only if the stated conditions hold. Then, δ_1 can be solved as either of $\pm\sqrt{\omega_{12}\omega_{13}/\omega_{23}}$, i.e., unique up to a sign change. It follows that $\delta_2 = \omega_{12}/\delta_1$, $\delta_3 = \omega_{13}/\delta_1$, $\sigma_2^2 = \omega_{22} - \delta_2^2$, and $\sigma_3^2 = \omega_{33} - \delta_3^2$ also exist as functions of ω_{ij} if and only if the stated conditions hold. ■

Under the conditions of this theorem, the likelihood ratio statistic in (15) is exactly zero for $n = 3$. Therefore, (15) is also useful to check whether the factor structure fits data for $n = 3$. If (15) yields a zero, the factor error structure is an equivalently alternative description of the reduced-form error term and is ideal for measuring the information contributions of markets.

II.4 Comparison with exiting price discovery measures

When an asset is traded in multiple markets, information content measures in the literature are largely based on the VECM in (2), with different treatments of the reduced-form shock vector ε_t. To see the differences, we write $\varepsilon_t = W z_t$, where W is a $n \times n$ matrix; z_t is a $n \times 1$ *structural shock* vector with zero mean and diagonal variance matrix. Authors typically aim to interpret the structural shock $z_{i,t}$ in z_t as the information contribution of market i. In general, $z_{i,t}$ may affect other markets contemporaneously. The efficient price change is decomposable as $\Delta m_t = h' W z_t = \sum_{i=1}^{n} h' W_i z_{i,t}$, where W_i is the ith column of W and $h = [\alpha_\perp' \Phi(1) \iota_n]^{-1} \alpha_\perp'$ is defined under (6). The information shares $(h'W_i)^2 \sigma_{z,i}^2 / \sum_{j=1}^{n} \left(h' W_j\right)^2 \sigma_{z,j}^2$, where $\sigma_{z,i}^2$ is the variance of $z_{i,t}$, are readily computed from this decomposition as the structural shocks in z_t are uncorrelated. The main issues within this framework are the identification of W and the identification of individual shocks in z_t, which need extra assumptions or restrictions.

The assumption used by Hasbrouck (1995) is $W = F$, where F is the lower-triangular Cholesky factor of Ω (such that $FF' = \Omega$) and the variance of z_t is I_n. This implies that the shock $z_{i,t}$ has no effect on market j for any $j < i$ at time t. In the case where there are no correlations amongst the shocks in ε_t, F becomes diagonal and this strategy will work well. However, in practical scenarios where the reduced-form shocks in ε_t are correlated, the information shares computed with $W = F$ depend on the order of the log prices placed in p_t. Hasbrouck (1995) suggests computing the information shares for all $n!$ Orderings of prices in p_t, and then finding the lower and upper bounds for each market's contribution.

A refinement to the approach of Hasbrouck (1995) is Grammig and Peter (2013), who use a result of Lanne and Lütkepohl (2010) to exploit the fact that W is identifiable (up to column permutations) when z_t follows a mixture of normal distributions. In this case, the variance of z_t is a weighted average of I_n and a diagonal matrix. To identify the shocks in z_i (or resolve column permutations of W), Grammig and Peter (2013) further assume that the self-effect of shock $z_{i,t}$ is positive ($w_{ii} > 0$) and greater than the cross-effects ($w_{ii} > |w_{ji}|$ for $j \neq i$), where w_{ji}'s are elements of W. The authors argue that these assumptions are practically plausible and useful for identifying market-specific shocks.

Another information contribution measure, also based on the VECM in (2), is known as component shares, see Booth, So and Tse (1999), and Harris, McInish and Wood (2002). The component shares are inspired by the permanent-transitory decomposition of Gonzalo and Granger (1995), where the log price p_t vector is decomposed into a permanent or I(1) component y_t and a transitory or I(0) component x_t: $p_t = A_1 y_t + A_2 x_t$, where $y_t = \alpha_\perp' p_t$. With $\alpha_{\perp,i}$ being the ith element in $\alpha_\perp$, the component share for market i is defined as $CS_i = \alpha_{\perp,i} / \sum_{j=1}^{n} \alpha_{\perp,j}$. Hasbrouck (2002) points out the y_t is generally not a martingale. Further, Yan and Zivot (2010) and Putnins (2013) suggest that a combined use of the component share and Hasbrouck (1995) information share can help sort out confounding effects of permanent and transitory shocks.

In addition to the VECM, De Jong and Schotman (2010), and Ozturk and van Dijk (2017) use unobserved component (UC) models to measure information contributions. The UC information shares are based on the contributions to the efficient price change of the deviations in $d_t = p_t - \iota_n m_{t-1}$. The main assumption in the UC model is that the variance matrix of d_t is diagonal, which permits decomposing the R-squared in the regression of Δm_t on d_t into the contributions of the elements in d_t. The authors argue that this assumption is more plausible than assuming the diagonality of Ω in the VECM.

The paradigm of this literature so far has been the belief that one can break up the efficient price change into exactly n market-specific shocks. We challenge this paradigm by arguing for the importance of common information in price formation and presenting empirical support (see Section 4). Our information share measures are based on a testable assumption (i.e., factor error structure) that is supported by our empirical analysis. They reflect the contributions to the variance of the efficient price by individual markets and the common shock, which existing measures are unable to attain.

III. Data and Summary Statistics

We use the same data as in Hasbrouck (2021a). We extract quote and trade prices of IBM from Oct 3 to Nov 11 of 2016 from WRDS Trade and Quote (TAQ) database. Unlike in Hasbrouck (2021a), each quote and trade has only one "exchange timestamp" in our data. All other aspects of our data are the same as in Hasbrouck (2021a). Our numbers of trades and quotes across exchanges are identical to Hasbrouck's Table 5. We construct returns between 9:45 am and 4 pm at 0.01, 0.1, 1, and 2 seconds. The 0.01 second interval is long enough to allow simultaneous price changes in multiple markets after an information event. Mid-quote returns are used when estimating information shares across exchanges. Trade returns are based on transaction price

Table 1 reports the daily percentage of periods with 0, 1, and 2 price changes at different sampling intervals. At 0.01 second, 99.2% of periods have no quote change and 99.6% have no change in trade price. At 10^{-5} second intervals, one would expect a tiny percentage of periods with price changes. It would require a large number of lags to capture return dynamics, say over 10 seconds. At 1 second intervals, the frequency of price change increases substantially to 32.5% (=16.6%+15.9%) for quotes and 28% (=21.2%+6.8%) for trades. It increases to over 45% at 2 second intervals for both quotes and trades. If price changes are proxies for information arrivals to the market, e.g. Du and Zhu (2017), information arrivals are negligible at sub-second intervals, and become noteworthy at 1 second intervals.

When prices do change, it is quite often that prices in different markets, of quotes and trades, change in the same period. Among periods with quote changes, the percentage of periods with 2 price changes is 0.21/(0.58+0.21) = 27% when sampled at 0.01 second. It increases to 34% at 0.1 second, 49% at 1 second, and 57% at 2 second. For trade price, the percentage of 2 price changes among periods of non-zero price changes is between 13 to 29%. This explains the relatively high return correlations across markets reported in Table 2. Across exchanges, return correlation increases from 0.465 at 0.01 second, to 0.617 at 0.1 second, to 0.757 at 1 second, and to 0.816 at 2 second intervals. At all sampling intervals, return correlations between quotes and trades are lower than those across exchanges. This is because there are 14 times more quotes than trades; Hasbrouck (2021a, Table 2). It is less likely to have changes in quotes and trade prices in the same period. Quotes are more evenly distributed across markets, with33% on NYSE and 67% on other exchanges; Hasbrouck (2021a, Table 4 Panel B). It is more like to have simultaneous quote changes in different markets.

Table 1: Frequency of Zero and Non-zero Returns

This table reports summary of the daily percentage of periods with 0, 1, and 2 non-zero returns at different sampling intervals.

Sampling Interval	0.01 second			0.1 second			1 second			2 seconds		
Non-zero Returns	0	1	2	0	1	2	0	1	2	0	1	2
Mid-quotes												
Average	99.2%	0.58%	0.21%	94.3%	3.73%	1.94%	67.6%	16.6%	15.9%	52.5%	20.6%	26.9%
St Dev	*0.2%*	*0.2%*	*0.1%*	*1.4%*	*1.0%*	*0.5%*	*5.8%*	*2.5%*	*3.7%*	*7.0%*	*2.3%*	*5.5%*
Min	*98.6%*	*0.4%*	*0.1%*	*90.5%*	*2.4%*	*1.0%*	*54.4%*	*11.3%*	*9.2%*	*37.7%*	*14.5%*	*16.8%*
Max	*99.5%*	*1.1%*	*0.3%*	*96.5%*	*6.7%*	*3.0%*	*77.7%*	*23.4%*	*22.2%*	*65.4%*	*25.1%*	*37.2%*
Trade Prices												
Average	99.6%	0.38%	0.06%	96.2%	3.10%	0.65%	72.1%	21.2%	6.8%	54.4%	32.4%	13.3%
St Dev	*0.1%*	*0.1%*	*0.0%*	*1.1%*	*0.9%*	*0.2%*	*5.2%*	*3.3%*	*2.1%*	*6.3%*	*3.1%*	*3.7%*
Min	*99.0%*	*0.3%*	*0.0%*	*91.9%*	*2.2%*	*0.4%*	*52.7%*	*16.9%*	*3.8%*	*32.3%*	*27.9%*	*7.9%*
Max	*99.7%*	*0.9%*	*0.1%*	*97.3%*	*6.6%*	*1.4%*	*78.4%*	*33.8%*	*13.5%*	*63.2%*	*43.0%*	*24.6%*

Table 2: Cross-market Return Correlations

This table reports summary of daily cross-market return correlations at different sampling intervals.

Sampling Interval	0.01 second	0.1 second	1 second	2 seconds
Mid-quotes (Listing vs Other)				
Average	0.465	0.617	0.757	0.816
St Dev	*0.051*	*0.049*	*0.048*	*0.045*
Min	*0.360*	*0.508*	*0.649*	*0.705*
Max	*0.586*	*0.734*	*0.855*	*0.899*
Quotes against Trades				
Average	0.246	0.341	0.438	0.501
St Dev	*0.047*	*0.061*	*0.078*	*0.088*
Min	*0.123*	*0.176*	*0.206*	*0.222*
Max	*0.307*	*0.416*	*0.534*	*0.612*

IV. Common and Market-specific Information Shares

This section reports the estimated common and market-specific information shares for IBM from Oct 3 to Nov 11, 2016, as in Hasbrouck (2021a). We have only one timestamp, therefore are unable to compare the information shares based on the participant time and the securities information process time. We estimate the information shares of the listing exchange and other exchanges. We also estimate the information shares of quotes and trade prices. The main finding is that the common information accounts for over 90% of the information flow at 1 and 2 second intervals. It dominates the idiosyncratic information shares of individual markets or price series at 0.01 and 0.1 second intervals.

Model specification and estimation

Theorem 1 requires n $\geq$ 3 for the 2n parameters in (10) to be identified from n(n+1)/2 equations in (9). Since n = 2 in our applications, we set $\delta_1 = \delta_2$ to achieve identification. This is reasonable when most investors trade on both markets.

The VECM in (2) is estimated daily for the four sampling intervals. Following Hasbrouck (2021a), the number of lags in the VECM is determined by 10 seconds, i.e. 1000 lags at 0.01 second, etc. The results are very similar when we use 30 seconds to determine the lag length. Because of the longer sampling intervals, the number of lags and coefficients are much smaller than those of Hasbrouck (2021a), avoiding the need for parameter restrictions and the use of bridging approximation. As in Hasbrouck (2021a), the standard errors of the CIS and MIS are based on the standard deviations of daily estimates.

Table 3 reports the numbers of lags, the number of parameters, and the percentage of significant coefficients. As the sampling intervals between 0.1 to 2 seconds, the percentage of significant coefficients is relatively stable between 33.3% to 35.9%. It reduces sharply to 22.4% at 0.01 second. It appears that model fit of the VECM deteriorates at ultra-high frequencies.

Common information shares

Table 4 reports the average common information shares between the listing and other exchanges, and between quotes and trade prices, from Oct 3 to Nov 11, 2016. The main feature is the high common information shares. At 1 and 2 second intervals, the CIS is 90 to 94% across exchanges, and 85 to 90% between quotes and trades. At sub-second intervals, the common information is more than 60% of the total information flow. The daily minimum CIS is 58 to 90% across exchanges and 38 to 64% between quotes and trades. The standard deviations of the daily CIS are small and the average CIS are significantly

Table 3: Lags, Parameters, and Significance

This table reports the numbers of lags and parameters, and the percentage of significant coefficients, at different sampling intervals on Oct 3, 2016. The significance level is 5%.

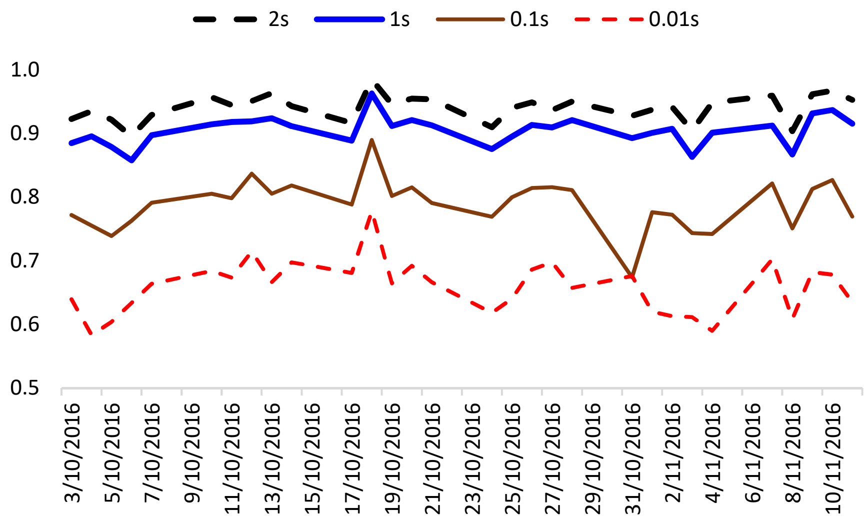
Sampling Interval	0.01 second	0.1 second	1 second	2 seconds
Lags	1000	100	10	5
Parameters	4002	402	42	22
% Significant	22.4	35.9	34.2	33.3

Table 4: Common Information Share

This table reports the common information shares between the listing and other exchanges, and between quotes and trade prices at different sampling intervals.

Sampling Interval	0.01 second	0.1 second	1 second	2 seconds
Listing and other exchanges				
Average	0.658	0.788	0.904	0.940
St Dev	*0.042*	*0.039*	*0.022*	*0.020*
Min	*0.583*	*0.674*	*0.857*	*0.895*
Max	*0.778*	*0.889*	*0.962*	*0.985*
Quotes and trades				
Average	0.594	0.748	0.858	0.899
St Dev	*0.082*	*0.084*	*0.076*	*0.076*
Min	*0.377*	*0.514*	*0.621*	*0.635*
Max	*0.702*	*0.863*	*0.963*	*0.993*

Figure 1: Daily CIS between Listing and Other Exchanges



greater than zero at 1%. For the CIS across exchanges, both the standard deviations and the Min-Max range decrease as the sampling interval increases. For the CIS between quotes and trades, both are stable across sampling intervals. The CIS is estimated from simultaneous price changes across markets, is directly related to the cross-market return correlations: the longer the sampling interval, the stronger the cross-market return correlation, the higher the CIS. That is indeed the case in Tables 2 and 4.

Table 4 shows that the CIS across exchanges are higher than the CIS between quotes and trades, especially at 0.01 second. This may reflect the greater frequency and price impact of public news than private information/beliefs transmitted via order flows. To the degree that quotes reflect more public news, quotes across different markets have similar arrival rates and greater common components. To the degree that trades reflect private demand for transactions, they are triggered by different events and often reflect different opinions or information. Therefore, trades occur less frequent and innovations in transaction prices are less correlated with innovations in quotes.

Figure 1 plots the daily CIS between listing and other exchanges at different sampling intervals. The plot for the daily CIS between quotes and trades is very similar. The daily CIS are highly correlated across all sampling intervals, but particularly for 1 and 2 second intervals. On Oct 18, all four CIS estimates has large increases, followed by large reversals. Both the CIS stability and high correlations are like the result of the high persistence in public information arrivals. The daily CIS at sub-second intervals have larger variations. Occasionally they move in opposite directions, e.g. around Oct 31, 2016.

Market-specific information shares

Table 5 reports the average information shares of the listing exchange (NYSE) and other exchanges over the sample period. After accounting for the common information across exchanges, the market-specific information shares are substantially lower than those reported by Hasbrouck (2021a), i.e. 40% for the NYSE and 60% for the other exchanges. At 0.01 to 1 second intervals, the information shares of other exchanges are slightly higher; but the differences are not significant at 5%. At 2 second intervals, the information share of the NYSE is statistically higher than that of the other exchanges; but both are numerically very small. At sub-second intervals Even at 0.01 second intervals, the sum of market-specific information shares is much lower than their common information shares in Table 4.

Table 6 reports the average information shares of quotes and trade prices. The striking result is that after accounting for the common information and the information content of quotes, the information share of trade prices is not significantly different from zero at 0.1 to 2 second intervals. The information

Table 5: Listing versus Other Exchanges

This table reports the average daily information shares of the listing and other exchanges, after accounting for their common information.

Sampling Interval	0.01 second	0.1 second	1 second	2 seconds
Listing exchange				
Average	0.160	0.099	0.051	0.036
St Dev	*0.038*	*0.032*	*0.019*	*0.017*
Min	*0.096*	*0.052*	*0.022*	*0.014*
Max	*0.239*	*0.172*	*0.108*	*0.083*
Other exchanges				
Average	0.181	0.111	0.044	0.024
St Dev	*0.050*	*0.035*	*0.017*	*0.013*
Min	*0.082*	*0.049*	*0.015*	*0.002*
Max	*0.272*	*0.179*	*0.088*	*0.060*
Test for Difference				
t stat	*-1.83*	*-1.35*	*1.49*	*3.32*

Table 6: Quotes versus Trade Prices

This table reports the average daily information shares of quotes and trade prices, after accounting for their common information.

Sampling Interval	0.01 second	0.1 second	1 second	2 seconds
Quotes				
Average	0.362	0.230	0.128	0.090
St Dev	*0.077*	*0.077*	*0.070*	*0.071*
Min	*0.184*	*0.102*	*0.036*	*0.003*
Max	*0.563*	*0.426*	*0.350*	*0.339*
Trades				
Average	0.043	0.022	0.014	0.011
St Dev	*0.029*	*0.022*	*0.017*	*0.016*
Min	*0.012*	*0.002*	*0.000*	*0.000*
Max	*0.126*	*0.089*	*0.070*	*0.063*
Test for Difference				
t stat	*21.44*	*14.08*	*8.71*	*5.90*

shares of quotes are relatively high and statistically significant at 0.1 and 1 second intervals. After accounting for the common information, trade prices have no independent information content. This is consistent with the high quote information share of 65% relative to trades (35%) reported by Hasbrouck (2021a). It provides direct support to central role of limit orders in price discovery documented by Brogaard et al. (2019), and their observation of "price discovery without trading".

Common information shares over time

The dominance of the common information deserves further attention. Economically, it reflects the price impact of public information flow, e.g. news and order flows, versus the price impact private information and belifs. Econometrically, it captures the high contemporanous return correlations across exchanges at different sampling intervals. Malceniece et al. (2019) show that the highly correlated trading strategies of HFT sharply increased return correlations across stocks. They may have also increased return correlations across exchanges, especially at time intervals beyond human reaction. The proliferation of HFT started after the SEC passed Reg NMS in 2005. To see how the CIS increased over time, we estimate the CIS between exchanges in 2000 before the HFT proliferation, and in 2008 at the start of the HFT proliferation. The sample period is October and early November of each year.

Figure 2 depicts the 1-second common and market-specific information shares in October and early November of 2000, 2008, and 2016. We see the average CIS has sharply increased over time. In 2000, trading was much slower: the few fully automated trading platforms at the time had execution time in the seconds (Haldane, 2010). Returns sampled at 1 second are sparse and have low cross-market correlations, resulting in the average CIS just 11.8%. The floor specialists at the NYSE dominated the pricing of IBM, and other exchanges had negligible information. Ignoring the common information across markets may not have a distorting effect on the estimation of market-specific information share. By 2008, HFT was highly profitable[2] and has begun to proliferate. HFT increases trading intensity and cross-market correlations. The average CIS increases to 67.1%, dominating the combined MIS.[3] The optic cable using dark fiber in a straight line between Chicago and New York became operational in 2010. HFT firms have continued to advance their trading technologies and strategies, further synchronizing price changes across markets. By 2016, the cross-market return correlation at 1-second interval is 0.757 (Table 2), and the average CIS has increased to 90.5% (Table 4).

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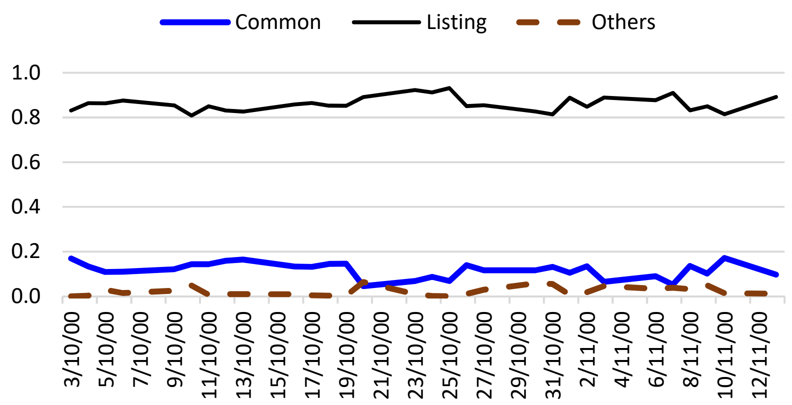
[2] Menkveld (2013) reports an estimated Sharpe ratio of 9.35 for an HFT firm trading in Europe in 2008.

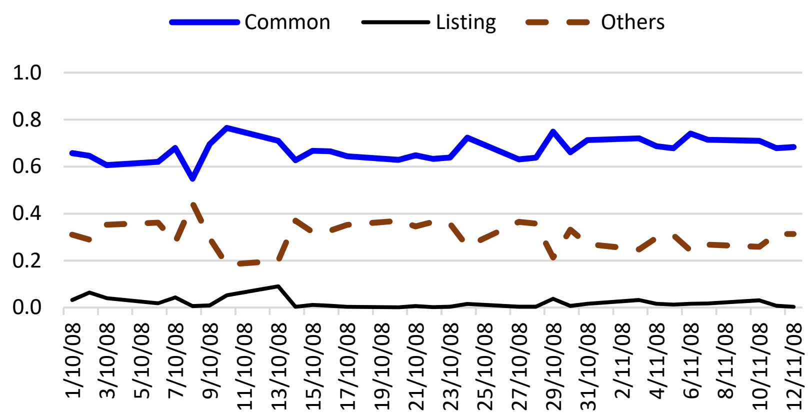
[3] The low MIS of the NYSE is consistent with the finding of Grammig and Peter (2018, Figure 3 panel B) that the NYSE has the lowest monthly information share in Oct-Nov 2008.

Figure 2: IBM Information Shares at 1-Second Interval

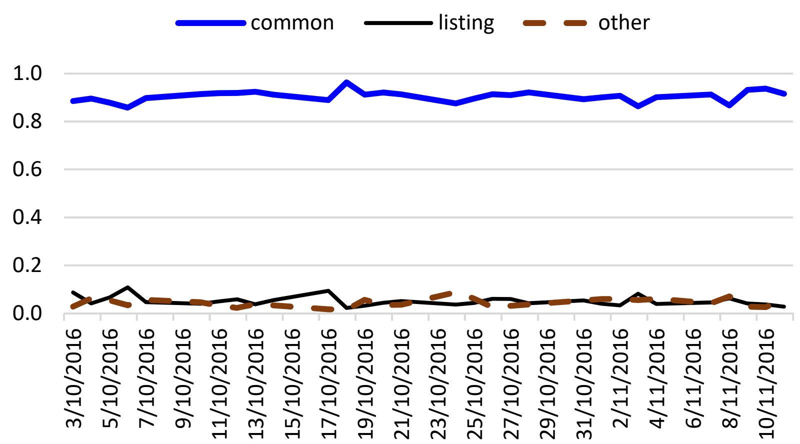
Panel A: October 2000



Panel B: October 2008



Panel C: October 2016



V. Conclusion

When a risky asset is traded in multiple markets, this study proposes a price discovery model that decomposes price innovations in different markets into a common component and a market-specific component. The separation of the cross-market common information overcomes the economic and econometric issues in the Hasbrouck model, e.g. the "who-moves-first" interpretation of price discovery, and the need for ultra-high frequency data to disentangle cross-market return correlations. We find that cross-market common information has increased with the proliferation of HFT. It dominates price discovery across sampling intervals at 0.01 second or longer. The common information share increases monotonically as the sampling interval increases. After accounting for the common information share, the market-specific information shares of the listing and other exchanges are not statistically different. Quote prices contain significant information but trade prices do not.

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